Optimum Conditions for the Cardio-Vascular-Pulmonary System Obtained in the Irreversible Finite Speed Thermodynamics Framework

II. Optimizing the net power delivered by the organism

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Starting from several simple hypotheses regarding the gas diffusion phenomena taking place through the semipermeable walls of the capillaries and regarding the energy and substance balance in the organism, we show that: 1) Although there is no blood speed to maximize the oxygen transported by the cardio-vascular system, there is a certain blood speed which maximizes the net power delivered by the organism. 2) Also, there exists an optimum number of capillaries maximizing this net power.

Keywords: gas diffusion, semipermeable walls, cardio-vascular system

In [15] we showed that the Oxygen flow transported by the cardio-vascular system always increases (in a non-linear fashion) with the blood speed: at low blood speeds the two are directly proportional, but for higher speeds the Oxygen flow tends to reach a plateau. This dependency has an effect on the organism's ability to convert the chemical energy of the ingested food into useful mechanical energy. In order to evaluate this effect we consider a simplified model of the energy interactions in a living organism (Figure 1): analogous to an automobile engine, the organism transforms the chemical energy of the fuel (food) and of the Oxygen into mechanical energy, with a certain efficiency (the process does not use heat as an intermediary, like a thermal machine, but the conversion is direct, like in a fuel cell); a part of the resulting mechanical energy is used for powering the "Oxygen delivery system" (analogous to the turbo-compressor which forces Oxygen into the automobile's engine), and the rest is useful energy available for the organism's needs; from the mechanical energy powering the pump (the heart's chambers), only a fraction becomes hydraulic energy, the rest being lost as heat because of friction and other internal irreversibilities of the heart; the effect of the hydraulic energy is the blood speed *w*, which determines the volumetric blood flow *D*, which in turn determines the Oxygen mass flow $\dot{m}$.

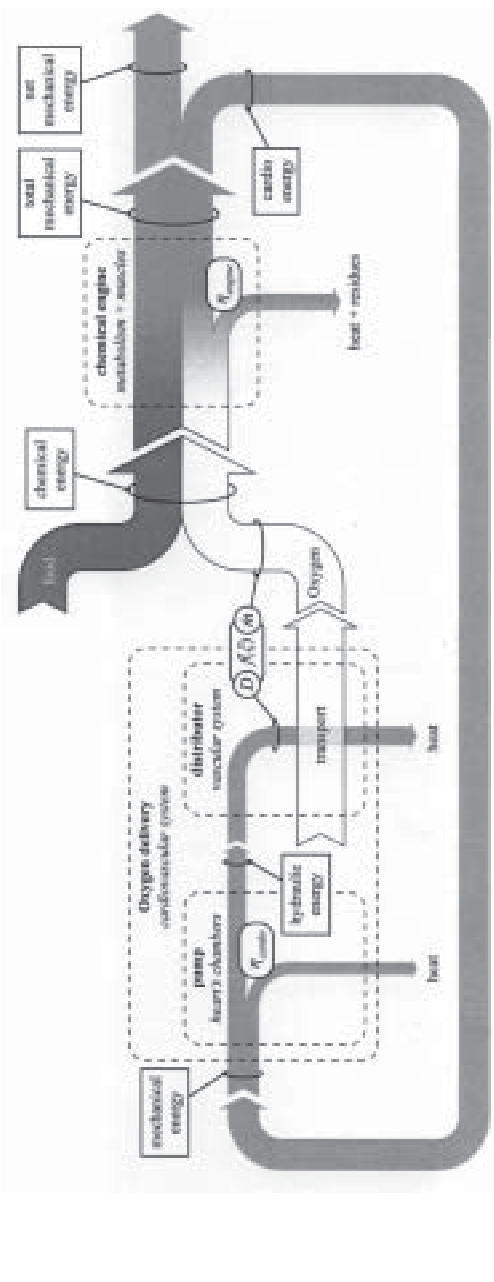

Fig. 1. Energy balance in the organism

The net power

We make the hypothesis that food and Oxygen combine in a certain "stoichiometric ratio". Considering that in the organism there is enough "fuel" (ingested food), it follows that the rate of chemical energy ’! mechanical energy conversion in muscles is determined by the Oxygen intake. In this case, we can approximate that the total mechanical power delivered by muscles is directly proportional to the Oxygen flow brought by the cardio-vascular system:

$$P_{total} = K_{muscle}\dot{m} \qquad (1)$$

From this power, a part is directed to the cardio-vascular system (being delivered by the myocardium and used by the heart), so the net power available for other activities is:

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$$P_{net} = P_{total} - P_{cardio} \qquad (2)$$

Considering that the total hydraulic resistance of the blood vessels is constant and equal to $R_{vascular} = \Delta p_{vascular}/D$ [7, 12], the power needed for pumping is proportional to the blood flow squared:

$$P_{pumping} = D\Delta p_{vascular} = D\left(R_{vascular}D\right) = R_{vascular}D^2 \qquad (3)$$

This pumping power is obtained with a certain efficiency from the mechanical power delivered by the heart (the efficiency η_{cardio}, of the heart regarded as a pump):

$$P_{pumping} = \eta_{cardio}P_{cardio}$$
$$P_{cardio} = \frac{R_{vascular}}{\eta_{cardio}}D^2 \qquad (4)$$

We introduce the reduced speed (defined in [15]):

$$P_{cardio} = \frac{R_{vascular}K_p^2S_{lateral}^2}{\eta_{cardio}}\xi^2 \qquad (5)$$

From [15] we know the Oxygen flow formula, so we can write the net power as a function of the reduced speed ξ:

$$P_{net} = \underbrace{K_{muscle}\,\underbrace{\dot{m}_{max}2\xi\frac{e^{\frac{1}{\xi}}-1}{e^{\frac{1}{\xi}}+1}}_{\dot{m}}}_{P_{total}} - \underbrace{\frac{R_{vascular}K_p^2S_{lateral}^2}{\eta_{cardio}}\xi^2}_{P_{cardio}} \qquad (6)$$

Besides the non-dimensional factors depending on ξ all the others represent powers. We denote:

$$P_{total_{max}} = K_{muscle}\dot{m}_{max} \qquad (7)$$

$$P_{cardio_2} = \frac{R_{vascular}K_p^2S_{lateral}^2}{\eta_{cardio}} \qquad (8)$$

$$B = \frac{P_{cardio_2}}{P_{total_{max}}} = 2\frac{R_{vascular}K_pS_{lateral}}{\eta_{cardio}K_{muscle}\Delta c_{HL}} \qquad (9)$$

to finally write the net power:

$$P_{net} = P_{total_{max}}g(\xi)\text{, where } g(\xi) = 2\xi\underbrace{\frac{e^{\frac{1}{\xi}}-1}{e^{\frac{1}{\xi}}+1}}_{f(\xi)} - B\xi^2 \qquad (10)$$

B is a non-dimensional parameter depending on the system's characteristics. Studying the function $g(\xi)$, we note that when ξ is small, the term $B\xi^2$ is also small and does not affect too much the net power. But then, as ξ increases, $B\xi^2$ increases faster than $f(\xi)$ (which tends asymptotically to 1) and decreases dramatically the net power. There is a value ξ_{max} for the reduced speed at which P_{net} becomes zero (the heart beats so fast that the organism's ability to generate mechanical power is used entirely for pumping blood, leaving no power available for other needs). Between $\xi = 0$ and ξ_{max} there exists a value ξ_{optim} which maximizes P_{net}, as it can be seen in figure 2.

It remains to be determined through further research if the blood speed in biological organisms comes close to this optimal value, or is limited by other considerations.

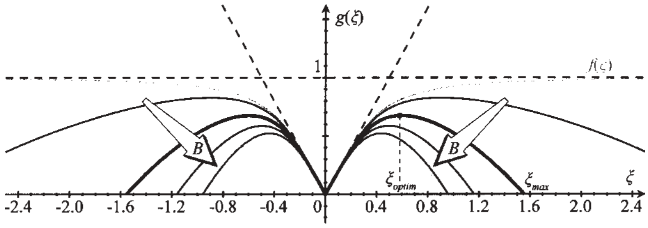

Fig. 2. Because of the power required by the heart, there is an optimum point.

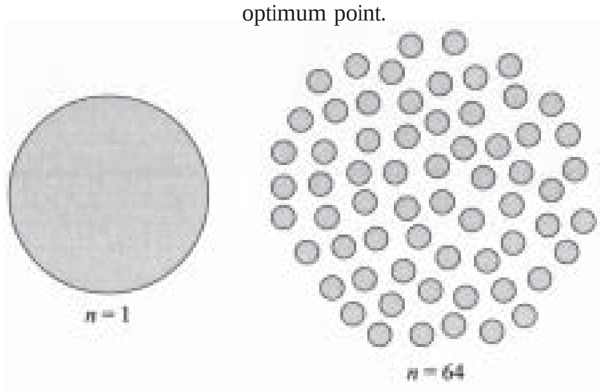

Fig. 3. The same cross-sectional area can be distributed to many thin tubes

The number of capillaries

The tube we considered does not have necessarily a circular cross-section. For a given area A of the cross-section, this shape could be divided in n smaller circles (fig. 3). This way the perimeter length $L_{perimeter}$ increases, and with it the lateral area $S_{lateral}$ increases too (this being the reason why the vascular system branches off in many thin capillaries).

Using the index 0 to mark the (hypothetical) situation when all the capillaries are united to form a single thick tube, we can write:

$$K_pS_{lateral} = K_pS_{lateral_0}\sqrt{n} \qquad (11)$$

$$\dot{m} = \Delta c_{HL}D\frac{e^{\frac{K_pS_{lateral_0}\sqrt{n}}{D}}-1}{e^{\frac{K_pS_{lateral_0}\sqrt{n}}{D}}+1} \qquad (12)$$

$$\dot{m} = \underbrace{\Delta c_{HL}D\frac{e^{\frac{1}{\xi_0}}-1}{e^{\frac{1}{\xi_0}}+1}}_{\dot{m}_0}\left(\frac{e^{\frac{1}{\xi_0}}+1}{e^{\frac{1}{\xi_0}}-1}\cdot\frac{e^{\frac{\sqrt{n}}{\xi_0}}-1}{e^{\frac{\sqrt{n}}{\xi_0}}+1}\right) \qquad (13)$$

But in the same time the number of capillaries also influences the hydraulic resistance $R_{vascular}$. From the Hagen-Poiseuille law [1], the hydraulic resistance of a tube of radius r is:

$$R_{tube} = \frac{8\eta}{\pi}\frac{L_{total}}{r^4}, \qquad (14)$$

where η is the fluid's dynamic viscosity. The total resistance of n tubes in parallel is:

$$R_{vascular} = \frac{1}{n}\frac{8\eta}{\pi}\frac{L_{total}}{r^4} \qquad (15)$$

The total cross-sectional area is $A = n\pi r^2$, so we can express the resistance as a function of A:

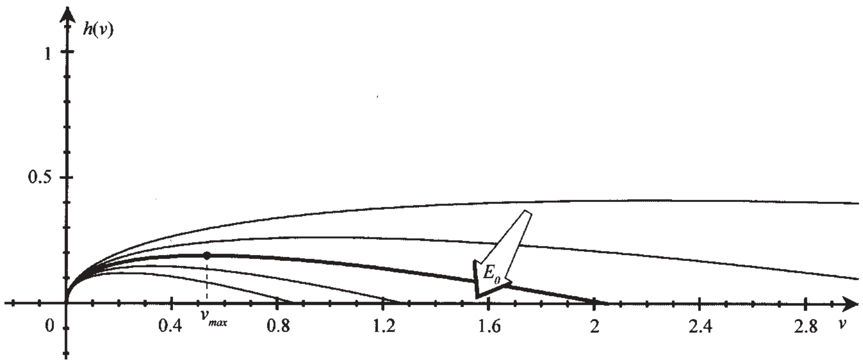

Fig. 4. Net power as a function of the number of capilaries

$$R_{vascular} = \underbrace{8\pi\eta \frac{L_{total}}{A^2}}_{R_{vascular_0}} n \tag{16}$$

So the mechanical power needed to pump the blood is directly proportional to the number of capillaries:

$$P_{cardio} = \underbrace{\frac{R_{vascular_0}}{\eta_{cardio}} D^2}_{P_{cardio_0}} n \tag{17}$$

With these, the net power as a function of the number of capillaries is:

$$P_{net} = \underbrace{K_{muscle}\dot{m}_0}_{P_{total_{\max,0}}} \left(\frac{e^{\frac{1}{\xi_0}}+1}{e^{\frac{1}{\xi_0}}-1} \cdot \frac{e^{\frac{\sqrt{n}}{\xi_0}}-1}{e^{\frac{\sqrt{n}}{\xi_0}}+1} \right) - P_{cardio_0} n \tag{18}$$

We make a variable change:

$$\nu = \frac{n}{\xi_0^2} \tag{19}$$

to be able to write:

$$P_{net} = P_{total_{\max,0}} \underbrace{\left(\frac{e^{\frac{1}{\xi_0}}+1}{e^{\frac{1}{\xi_0}}-1} \right)}_{C_0} \cdot \underbrace{\left[\frac{e^{\sqrt{\nu}}-1}{e^{\sqrt{\nu}}+1} - \underbrace{\left(\frac{P_{cardio_0}\xi_0^2}{P_{total_{\max,0}}} \frac{e^{\frac{1}{\xi_0}}-1}{e^{\frac{1}{\xi_0}}+1} \right)}_{E_0} \nu \right]}_{h(\nu)} \tag{20}$$

$$P_{net} = P_{total_{\max,0}} C_0 h(\nu), \text{ where } h(\nu) = \frac{e^{\sqrt{\nu}}-1}{e^{\sqrt{\nu}}+1} - E_0\nu \tag{21}$$

C_0 and E_0 are non-dimensional parameters depending on all the characteristics of the system (including the blood speed, which is considered fixed) and refer to the situation when all the capillaries are united in a single thick tube. The function $h(\nu)$ is also non-dimensional, capturing the dependence between the net power and the variable ν (which is directly proportional to the number of capillaries n). This dependency looks like in figure 4.

We note that there exists an optimal number of capillaries: too few capillaries and the gas exchange will be insufficient, too many capillaries and the hydraulic resistance will increase so much that the power needed for pumping will reduce the remaining power available for the rest of the organism.

Conclusions

According to a simple, yet general thermodynamic model of the energy and substance fluxes in which the cardio-vascular system is involved, accounting for the power needed by the heart to push blood through the vascular system, the blood speed can be optimized to maximize the net power produced by the organism. But because there may be other limiting factors (e.g., the mechanical strength of the blood vessels), further research is needed to see if in reality the biological organisms come close to this optimum point.

Dividing the cross-sectional area of the thick blood vessels into many thin capillaries is beneficial to increasing the gas diffusion area, but also increases the pumping power. Between these two tendencies there exists a balance point, which determines the optimum number of capillaries (that which maximizes the net power delivered by the organism).

All these conclusions where obtained with the hypothesis that the various efficiencies (of the heart as a pump, of the muscles as motors etc.) do not depend on speed. It remains to be researched if the found optimum points are influenced by the phenomenon of efficiencies decreasing at higher blood speeds. This study can be performed using the Finite Speed Thermodynamics [13], analyzing in detail various kinds of irreversibilities due to the finite speed of the processes in the heart, the lungs, the internal organs, the brain and the muscles.

Because most of the variable parameters can be easily measured, the study can be extended by actual pulse, arterial/venous pressure and Oxygen saturation measurements in various physical effort regimes. However, for such measurements to be relevant, one needs to account for the influence of the baroreflex regulatory system, which was not included in the present model. Because even the baroreflex system works with finite speed, it may be possible to treat its processes in a unified manner, correlated with the processes in the cardio-vascular-pulmonary system (also in FST, but extended to the domain of electrochemical information exchange in the whole organism).

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Nomenclature

0 (as index) refers to the hypothetical situation when all the capillaries are united into one single thick tube

A [m^2] = cross-sectional area of a tube

B (non-dimensional) = parameter depending on the system's characteristics (9)

Δc_{HL} [kg/m^3] = difference between gas mass concentration in the outside environment and in the intercellular environment

C_0 (non-dimensional) = parameter depending on the system's characteristics (20)

D [m^3/s] = fluid (blood) volumetric flow
e (non-dimensional) = natural logarithms' base
E_0 (non-dimensional) = parameter depending on the system's characteristics (20)
(ξ) (non-dimensional) = function capturing the dependency between the Oxygen flow and the blood's reduced speed
$g(\xi)$ (non-dimensional) = function capturing the dependency between the net power and the blood's reduced speed (10)
γ [m^{-1}] = form factor of a tube
$h(\nu)$ (non-dimensional) = function capturing the dependency between the net power and the number of capillaries (21)
H (as index) refers to pulmonary capillaries
K_p [m/s] = gas permeability through a semipermeable membrane
K_{muscle} [J/kg] = proportionality coefficient between the mechanical power produced by the organism and the Oxygen flux needed to produce it (1)
L (as index) refers to systemic capillaries
$L_{perimeter}$ [m] = circumference of a tube
L_{total} [m] = length of a tube
$\dot{m}$ [kg/s] = gas mass flow entering a tube through its walls
$\dot{m}_{\max}$ [kg/s] = maximum Oxygen flow that can be transported by the cardio-vascular system (when there is no blood and the semipermeable membranes are adjoined)
n (non-dimensional) = number of capillaries
ν (non-dimensional) = variable directly proportional to the number of capillaries (19)
$\Delta p_{vascular}$ [N/m^2] = total pressure drop over the vascular system
P_{cardio} [W] = mechanical power of the heart
P_{pump} [W] = hydraulic power of the heart
P_{total} [W] = total mechanical power produced by the organism
P_{net} [W] = net mechanical power delivered by the organism
π (non-dimensional) = ratio between the circumference and diameter of any circle
r [m] = radius of a tube
$R_{vascular}$ [Pa'"s/m^3] = total hydraulic resistance of the vascular system
$S_{lateral}$ [m^2] = lateral surface area
η [Pa . s] = dynamic viscosity (from the Hagen-Poiseuille law)
η_{cardio} (non-dimensional) = hydraulic efficiency of the heart
ξ(non-dimensional) = fluid's reduced speed

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